Doing Mathematics Discussion — Emily Staskin

Course: Geometry

VA SOLs: G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the a) sum of the interior and/or exterior angles.

Learning Targets:

- I can determine the sum of the interior angles of a convex polygon.

Key Vocabulary: convex polygon, triangle, quadrilateral, pentagon, hexagon, … , interior angle, convex regular polygon

Materials: campus map picture, graph paper, blank paper, compasses, protractors

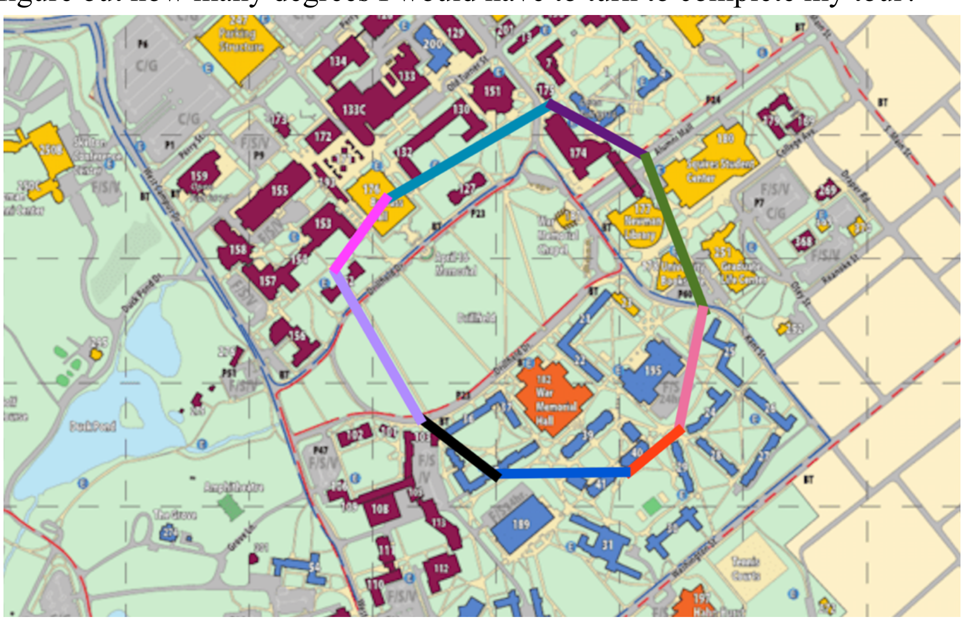
Total Time: 60 minutes

Engage (5min**):**

The teacher will begin the lesson by introducing a scenario to the students to think about:

"When I was in college, as many of you know, I gave tours of Virginia Tech's campus. This map shows a brief sketch of some of the places around campus that I liked to show to visitors. I start at Burruss Hall (*show where that is on the map*) and traveled clockwise (*show direction on map*) around the loop.

How can I figure out how many degrees I would have to turn to complete my tour?"



Key Questions:
- How can you describe the tour route? How many turns do I need to make?
- What is a polygon? What characteristics does it have? What is a convex polygon?

Explore (20min**):**

Before students begin the investigation, the teacher will make sure to have a conversation with them about the following questions if they were not discussed in the engagement: What is a polygon? What characteristics does it have? What is a convex polygon?

Then, using the materials provided for them on the table, students will be asked to look for patterns for the sum of interior angles of convex polygons.

General key questions:
- *What do you know about the interior angles of convex polygons?*
- *Do you think the sum of the interior angles will be the same for each variation of a quadrilateral, pentagon, hexagon, etc.? Why or why not? How can you test your conjecture?*
- *Can you organize your thinking in a chart or picture?*
- *What patterns do you notice between the sum of the interior angles of the convex polygons?*

While students are exploring on their own, the teacher will be monitoring their progress to determine which student solutions should be shared with the entire class and in what order. Some possible student solutions are below: (*The order the solutions are listed is the order the teacher plans on having the solutions shared with the class.*)

(1) Students may use their prior knowledge of the sums of the interior angles of triangles (180°) and quadrilaterals (360°) to determine that with each additional side of the convex polygon, the sum of the interior angles will increase by 180°.

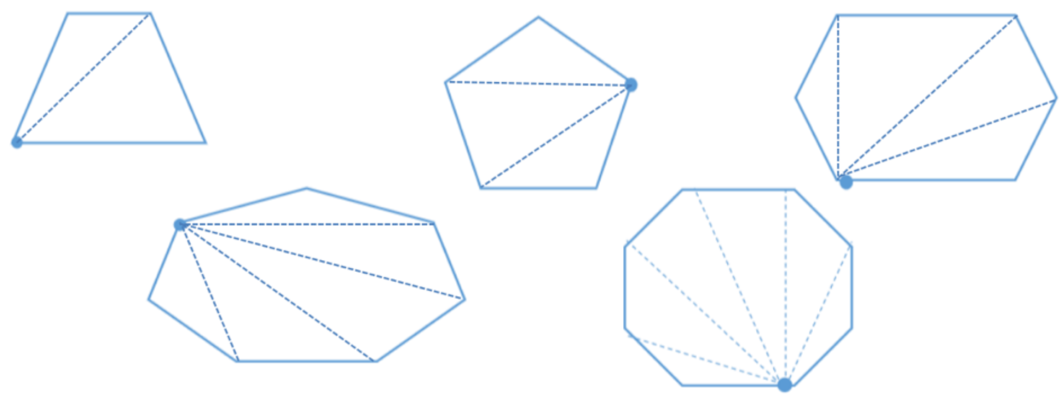
- *Are you convinced that your conjecture will work in all cases? How can you check/be sure?*

(2) Students may record the sums in a table. With every extra side, there is 180° added to the total sum. Students will most likely draw out convex polygons on their paper and measure the interior angles one at a time, then sum them to find the total sum.

Convex Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon
Number of Sides	3	4	5	6	7	8	9
Sum of Interior Angles	180°	360°	540°	720°	900°	1080°	1260°

- *What patterns do you notice within your table?*
- *How can you be sure that the sum of the interior angles is the same for all quadrilaterals, pentagons, hexagons, etc.?*
- *Can you represent your data in another way?*

(3) Students may be able to recognize that they can use their prior knowledge of the sum of the interior angles of a triangle to find patterns. In this method, students will partition each convex polygon into triangles that do not overlap. Then, the sum of the interior angles will be 180° times the number of triangles in the convex polygon.



- *How can you be sure that each triangle accounts for 180° of the total sum of interior angles? Can you show me where this 180° comes from?*
- *How can you extend your thinking to find the sum of the interior angles of a convex polygon with n sides?*

(4) Students may use any of the previous solution strategies to then write a formula for the sum of the interior angles based on *n*, the number of sides of the convex polygon. The formula is *180(n-2)*.

- *What methods did you use when you started this problem?*
- *What patterns did you notice?*
- *How does your formula represent those patterns? Where are they seen in the formula?*

Explain (15min**):**

The teacher will select students to share out their solution strategies (in the order listed above). If there is more than one variation of the same solution strategy from above, the teacher may ask both students to present to compare their representations. Students will be encouraged to explain their thinking and make connections between the various solutions strategies.

- *What assumptions were made in this solution strategy?*
- *What similarities or differences do you notice among representations?*
- *How can each representation help us to notice the patterns in the sums of the interior angles of various convex polygons?*

If none of the students generate the third solution (using triangles to partition the convex polygon), the teacher will then ask some questions to prompt students to explore the problem from this perspective. Students will be given time to further explore this method before the teacher continues the explanation.

- *How can you use your prior knowledge of the sum of the interior angles of a triangle to help you with other convex polygons?*
- *Can you represent the convex polygons as multiple triangles? How?*

Once the students have all had a chance to explore partitioning the convex polygons into triangles, the teacher will use this representation to connect back to the formula (or help the students derive the formula if no one has done so yet).

The teacher will focus on where each aspect of the formula comes from in the students' work with partitioning convex polygons into triangles. The formula is $180(n-2)$ where n is the number of sides of the convex polygon. All of the sums of the interior angles are multiples of 180, hence multiplying 180 in the formula. In addition, there are $n-2$ triangles used to partition the convex polygon. Another way to think about this is looking at the vertices of the convex polygon. There are two out of the n vertices that are not used when partitioning the convex polygon into triangles. We multiply $n-2$ by 180 because for each triangle in the convex polygon, there are 180° accounted for.

- *What connections/patterns do you notice between the number of triangles in the convex polygon and the sum of the interior angles?*
- *What does the 180 represent in the formula?*
- *What does the n-2 represent in the formula? Why do we subtract 2 from n?*

Elaborate/Extend (10min**):**

The teacher will give students an additional problem to work on, that builds off of their knowledge from the previous exploration. Students will need to make connections to their previous work to solve this problem:

A convex "regular" polygon is one that has all the sides and all the angles the same size. Knowing this, if you have a convex regular polygon with 7 sides, how could you find the measure of just ONE angle?

- *What information were you given? What information do you need to know?*
- *What connections can you make to the previous activity?*
- *How can we relate the sum of the interior angles to just ONE angle in the convex regular polygon?*

Evaluate (10min**):**

At the end of class, students will be asked to explain in their own words the formula for the sum of the interior angles of a convex polygon. They will need to explain each component of the formula. This explanation can include any of the representations used in class, as long as the students show evidence of their thinking and understanding of the formula.

Accommodations for ELs:
Students will be encouraged to use pictures to explain their thinking. The final evaluation can be mainly pictorial explanations to explain the formula. The teacher will make sure to use pictorial representations in the explanation to make sure that the material is accessible to them.

Accommodations for ADHD:
No additional accommodations are needed because this is a hands-on activity and the students will be moving around and talking with one another during the majority of the class.

Accommodations for Gifted:
This task is set up to be accessible to students at a low level, while providing the opportunity for students to be challenged. Teachers will give these students much less guided questions during their exploration and encourage them to represent their thinking in another representation if they finish early. They can also spend more time individually making connections among their representations.

Accommodations for Struggling Learners:
Students who need extra support will be provided with a table to help organize their thoughts from the exploration and the discussion of partitioning the convex polygons into triangles. This will also be a good reference for them when the students are taking notes on the formula for determining the sum of the interior angles of the convex polygon.

Polygon	Number of Sides	Number of Angles	Number of Triangles	Sum of angles in one triangle	Sum of angles in this polygon
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
…..					
N-gon					